Subject: Leaving Certificate Maths

Teacher: Mr Murphy

Lesson 9: Trigonometry

9.1 Learning Intentions

After this week's lesson you will be able to;

- Apply ratio to trigonometry to solve problems
- Calculate the area of a triangle in a number of different ways
- Apply the Cosine and Sine rules to solve problems
- Derive some of the trigonometric formulae

9.2 Specification

2.3 Trigonometry	– use of the theorem of Pythagoras to solve problems (2D only) – use trigonometry to calculate the area of a triangle – solve problems using the sine and cosine rules (2D) – define sin θ and cos θ for all values of θ – define tan θ – solve problems involving the area of a sector of a circle and the length of an arc – work with trigonometric ratios in surd form	– use trigonometry to solve problems in 3D – graph the trigonometric functions sine, cosine, tangent – graph trigonometric functions of type • $f(\boldsymbol{\theta}) = a+b\text{Sin } c\boldsymbol{\theta}$ • $g(\boldsymbol{\theta}) = a+b\text{Cos } c\boldsymbol{\theta}$ for $a,b,c \in \mathbf{R}$ – solve trigonometric equations such as Sin $n\boldsymbol{\theta}$=0 and Cos $n\boldsymbol{\theta}$= ½ giving all solutions – use the radian measure of angles – derive the trigonometric formulae 1, 2, 3, 4, 5, 6, 7, 9 (see appendix) – apply the trigonometric formulae 1-24 (see appendix)

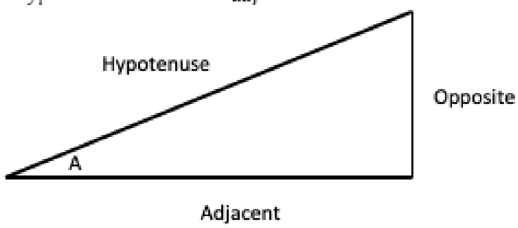
9.3 Chief Examiner's Report

In many topics, including coordinate geometry and trigonometry, drawing sketches or diagram may aid candidate in understanding how to solve the problem.

9.4 Trigonometric Ratios

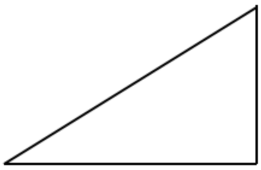
These ratios are used to help identify unknown sides and angles. The three ratios we use are known as Sine, Cosine and Tangent.
These ratios are **only** to be used with a **right**-angled **triangle.**

$Sin\ A = \frac{opp}{hyp}$ $\quad Cos\ A = \frac{adj}{hyp}$ $\quad Tan\ A = \frac{opp}{adj}$

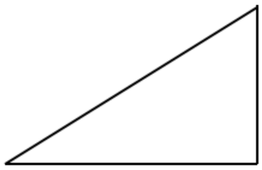


Below, copy down the two examples from the video:

Finding Side:



Finding Angle:



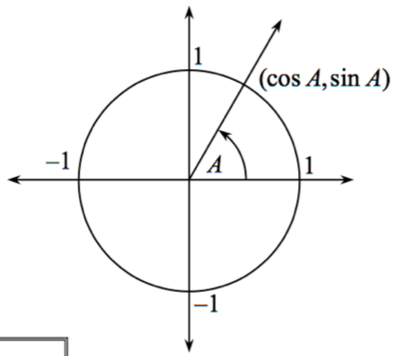
9.5 Tables Book

Trigonometry

Definitions

$\tan A = \frac{\sin A}{\cos A}$ $\quad \cot A = \frac{1}{\tan A}$

$\sec A = \frac{1}{\cos A}$ $\quad \operatorname{cosec} A = \frac{1}{\sin A}$



Trigonometric ratios of certain angles

A (degrees)	0°	90°	180°	270°	30°	45°	60°
A (radians)	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\cos A$	1	0	−1	0	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\sin A$	0	1	0	−1	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\tan A$	0	not defined	0	not defined	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

Basic identities

$\cos^2 A + \sin^2 A = 1$

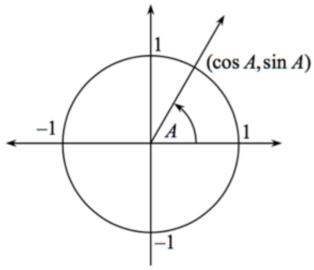
$\cos(-A) = \cos A$

$\sin(-A) = -\sin A$

$\tan(-A) = -\tan A$

9.6 Calculating Sin, Cos & Tan

Add in labels on the below unit circle:



We can use this circle to help calculate various values for the ratios. Below copy down the two examples from the video:

Sin 330°

Cos 590°

9.7 Trigonometric Equations

This are equations that allow us to establish a value for the angle when the value of the ratio is known. Sometimes the answer can be greater than 360°, in that case the limit will be explicitly mentioned in the question.

Examples:

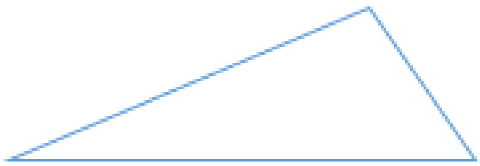
Sin $\theta = \frac{1}{2}$, $0° \leq \theta \leq 360°$

Tan $\theta = -\frac{1}{\sqrt{3}}, \ 0° \leq \theta \leq 720°$

9.8 Area of a Triangle

The formula from a trigonometry perspective is based on the same idea from geometry:

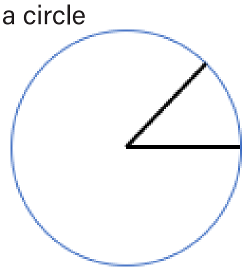
$$Area = \frac{1}{2}a.b.Sin\ C$$



C must be the angle between the two sides a and b.

9.9 Length of an Arc

An arc is a portion of the circumference of a circle



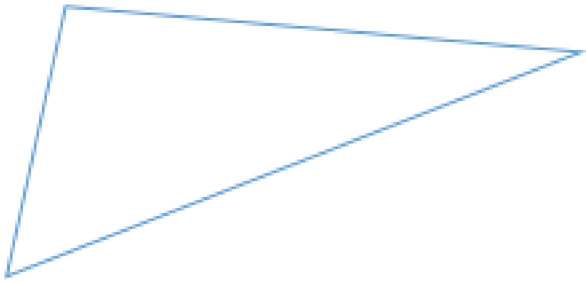
$l = r\theta$...Where θ is in radians

Example from the video:

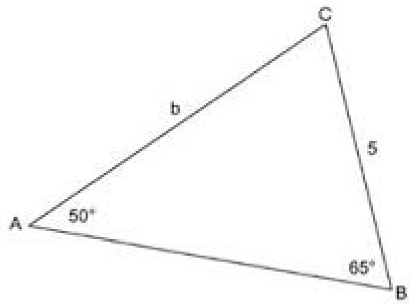
9.10 Sine Rule

This is a rule based on the Sine ratio within a triangle. Unlike the ratios however, this rule can be used with any triangle, right angled or not.

$$\frac{a}{Sin\ A} = \frac{b}{Sin\ B} = \frac{c}{Sin\ C} \quad \text{or} \quad \frac{Sin\ A}{a} = \frac{Sin\ B}{b} = \frac{Sin\ C}{c}$$



Example from the video:



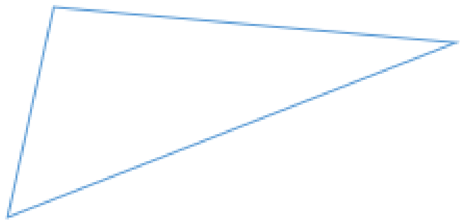
There is however an issue with this rule. As Sine of an angle is positive in both the 1st and 2nd quadrants of the unit circle, there can be a situation where there are two such triangles that satisfy the one Sine rule equation.

To reduce the risk of this happening, we use a different rule when possible:

Cosine Rule

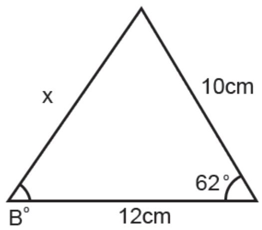
9.11 Cosine Rule

$a^2 = b^2 + c^2 - 2bc.\cos A$



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

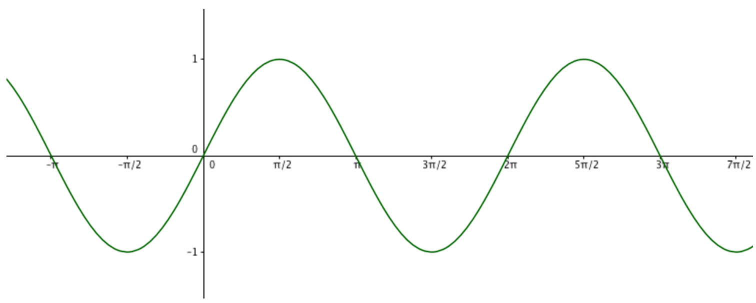
Example from Video:



9.12 Trigonometric Functions

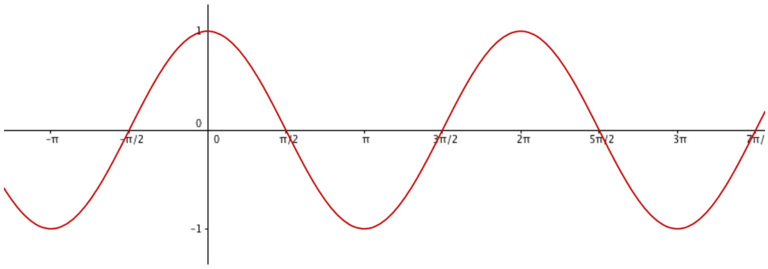
You can be asked to graph various Trigonometric functions as well as match functions to their graph. For this I would urge you to use a graphing software to see what transformations occur when you change the values in the function.

Sin θ

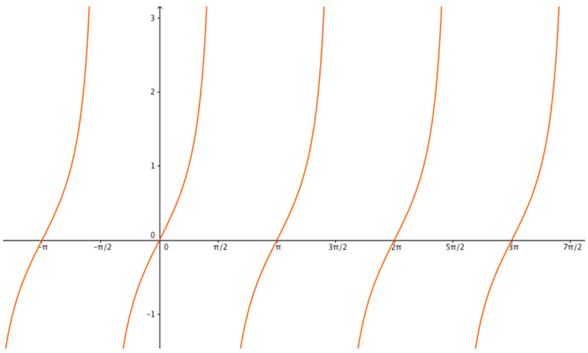


9.12 Trigonometric Functions

Cos θ



Tan θ

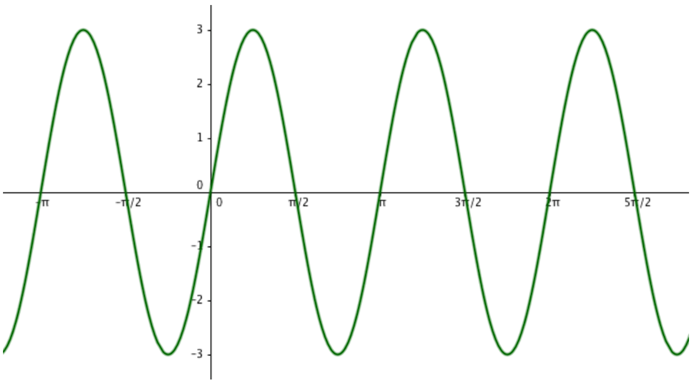


The above are the basic 3 graphs however you need to be able to identify graphs of the form:

$$f(x) = aSin(nx)$$

Where a affects the amplitude (distance from the x-axis), n affects the period, width of the graphs repeating pattern.

$$f(x) = 3\,Sin(2x)$$



Notice the increase in amplitude, from 1 to 3. Whereas, the period has decreased by half.

These are the 24 formulae present in the Tables book that allow you to manipulate various expressions. For the 1st 9 of these there is a proof required:

Basic identities

$\cos^2 A + \sin^2 A = 1$

$\cos(-A) = \cos A$

$\sin(-A) = -\sin A$

$\tan(-A) = -\tan A$

6. $\cos 2A = \cos^2 A - \sin^2 A$

7. $\sin(A+B) = \sin A \cos B + \cos A \sin B$

8. $\sin(A-B) = \sin A \cos B - \cos A \sin B$

9. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

Below are the full 24 as they appear in the tables book:

Basic identities

$\cos^2 A + \sin^2 A = 1$

$\cos(-A) = \cos A$

$\sin(-A) = -\sin A$

$\tan(-A) = -\tan A$

Products to sums and differences

$2\cos A \cos B = \cos(A+B) + \cos(A-B)$

$2\sin A \cos B = \sin(A+B) + \sin(A-B)$

$2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$2\cos A \sin B = \sin(A+B) - \sin(A-B)$

Sums and differences to products

$\cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$

$\cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$

$\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$

$\sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$

Compound angle formulae

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

Double angle formulae

$\cos 2A = \cos^2 A - \sin^2 A$

$\sin 2A = 2\sin A \cos A$

$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

$\cos 2A = \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$

$\sin 2A = \dfrac{2\tan A}{1 + \tan^2 A}$

$\cos^2 A = \frac{1}{2}(1 + \cos 2A)$

$\sin^2 A = \frac{1}{2}(1 - \cos 2A)$

9.13 Recap of the Learning Intentions

After this week's lesson you will be able to;

- Apply ratio to trigonometry to solve problems
- Calculate the area of a triangle in a number of different ways
- Apply the Cosine and Sine rules to solve problems
- Derive some of the trigonometric formulae

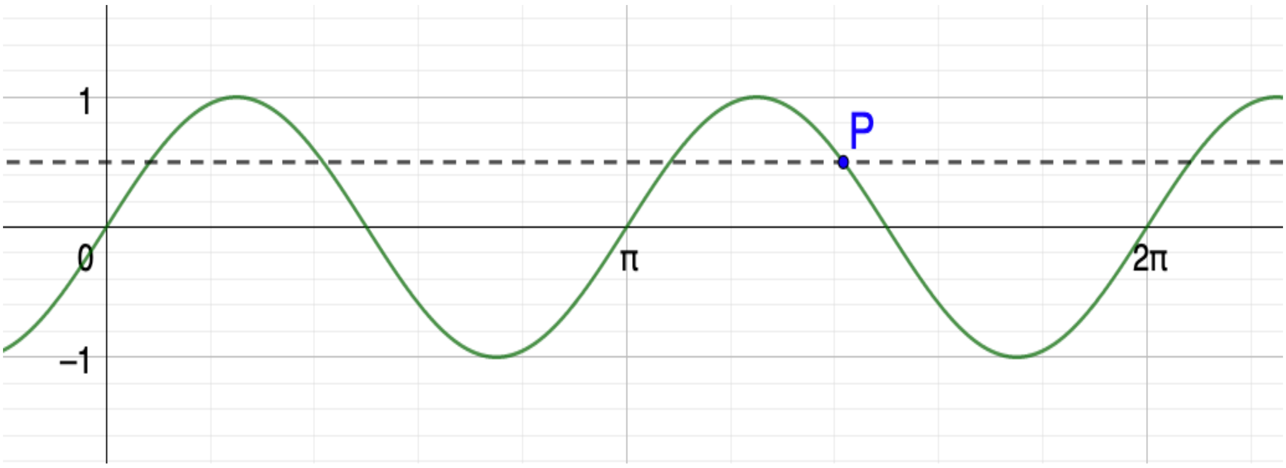
9.13 Recap of the Learning Intentions

Find all values of x for which $\cos(2x) = -\frac{\sqrt{3}}{2}$, *where* $0° \leq x \leq 360°$

The diagram below shows the graph of the function $f(x) = \sin 2x$. The line $2y = 1$ is also shown. One the same diagram sketch the graphs of $g(x) = \sin x$ and $h(x) = 3\ \sin 2x$.

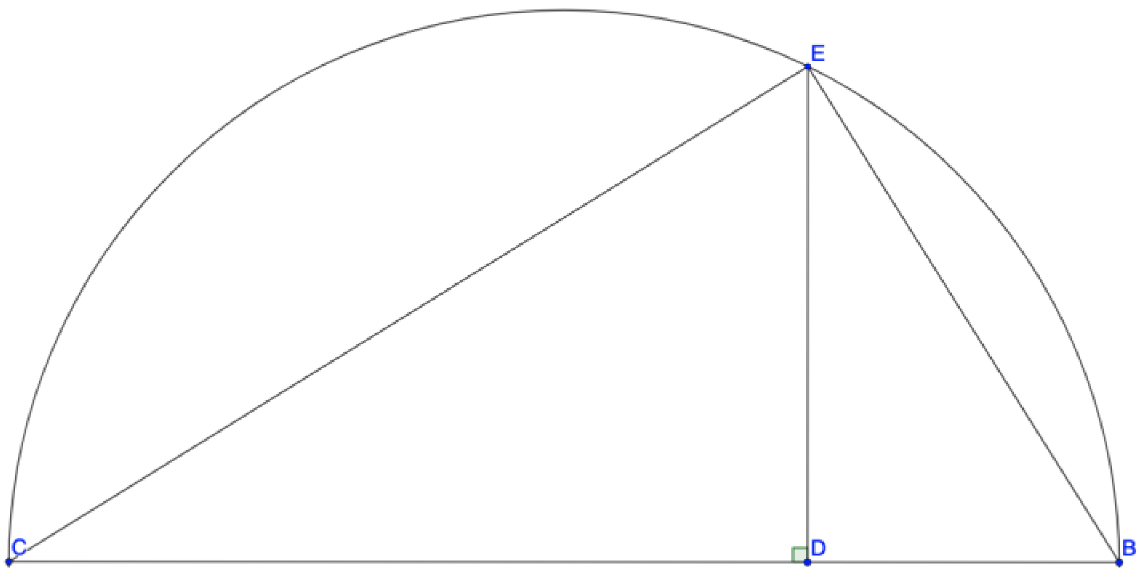
i) Label each function clearly.

ii) Find the coordinates of the point P



9.15 Solutions to 8.10

The diagram shows a semi-circle standing on a diameter [CB], and [ED] ⊥[CB]. Prove that the triangles ABD and DBC are similar.



$|\angle CDE| = |\angle BDE| = 90°$

$|\angle DEB| + |\angle DBE| = 90°$ angles in a triangle sum to 180°

$|\angle CDE| + |\angle DEB| = 90°$ angle in a semicircle = 90°

$|\angle DEB| + |\angle DBE| = |\angle CDE| + |\angle DEB|$ we've already said these pairs are 90° in steps 2 and 3

$|\angle CED| = |\angle DBE|$ angles in a triangle sum to 180°

As all three angles are the same, the triangles are equiangular and thus similar.